Jacobians

1. If $u=\frac{yz}{x}, v=\frac{xz}{y}, w=\frac{xy}{z}$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. 2. If $u=x+2y+z, v=x+2y+3z, w=2x+3y+5z$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.
3. If $u=x+y+z, v=xy+yz+xz, w=xyz$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. 4. If $u=x+y+z, v=x^2+y^2+z^2, w=xyz$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.
5. If $u=(1-x), v=x(1-y), w=xy(1-z)$ find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$. 6.If $x=uv, y=\frac{u+v}{u-v}$, find $\frac{\partial(u,v)}{\partial(x,y)}$.
7. If u = r sinθ cosφ, v = r sinθ sinφ, w = r cosθ find $\frac{\partial(u,v,w)}{\partial(r,\theta,\varphi)}$. 8. If $x=u, y=u\tan v,\ z=w$ show that $J.J'=1$.
9. If $x=\sqrt{vw}, y=\sqrt{uw}, z=\sqrt{uv}$ find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$. 10. If $x=e^v\sec u, y=e^v\tan u$ then evaluate $\frac{\partial(x,y)}{\partial(u,v)}$.
11. If $u=2axy, v=a(x^2-y^2)$ where $x=r\cos\theta,\ y=r\sin\theta$, then prove that $\frac{\partial(u,v)}{\partial(r,\theta)}=-4a^2r^3$..
12. If $x=\sqrt{vw}, y=\sqrt{uw}, z=\sqrt{uv}$and u = r sinθ cosφ, v = r sinθ sinφ, w = r cosθ find $\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)}$.
13. If $u_1=x_1+x_2+x_3, u_1^2u_2=x_2+x_3$ and $u_1^3u_3=x_3$, then find $\frac{\partial(u_1,u_2,u_3)}{\partial(x_1,x_2,x_3)}$.
14. If $u^3+v^3+w^3=x+y+z, u^2+v^2+w^2=x^3+y^3+z^3$ and $u+v+w=x^2+y^2+z^2$, then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.
15. If u, v, w are the roots of the equation $(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3=0$, in λ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.
16. If $u^3+v^3=x+y, u^2+v^2=x^3+y^3$ find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$. 17.If $u=x+y+z, uv=y+z, uvw=z$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.
18. If $u=x+y+z, v=xy+yz+xz, w=x^2+y^2+z^2$show that u, v, w are not independent and find relation.
19. Show that $u=\sin^{-1}x+\sin^{-1}y$ and $v=x\sqrt{(1-y^2)}+y\sqrt{(1-x^2)}$ are functionally dependent & find relation.
20. If $u=x+y-z, v=x-y+z, w=x^2+y^2+z^2-2yz$ show that u, v, w are not independent and find relation.
21. Show that $u=\tan^{-1}x+\tan^{-1}y, v=\frac{x+y}{1-xy}$ are functionally dependent & find relation.
22. If $u=3x+2y-z, v=x-2y+z, w=x(x+2y-z)$ show that u, v, w are not independent and find relation.
23. $u=x+2y+z, v=x-2y+3z, w=2xy-xz+4yz-2z^2$ show that u, v, w are not independent and find relation.
24. If $u=\frac{x+y}{z}, v=\frac{y+z}{x}, w=\frac{y(x+y+z)}{xz}$, then show that u, v, w are not independent and find the relation.
25. If $u=x+y+z, v=x^2+y^2+z^2, w=x^3+y^3+z^3-3xyz$ show that u, v, w are not independent and find the relation.
26. Find functional dependence and relation (i) $u=\frac{x+y}{x-y}, v=\frac{xy}{(x-y)^2}$ (ii) $u=\frac{x-y}{x+y}, v=\frac{x+y}{x}$ (iii) $u=\frac{x-y}{x+y}, v=\frac{xy}{(x+y)^2}$.

Taylor's Series

1. Obtain the expansion of $e^x \cos y$ in the neighborhood of $(1, \pi/4)$ by Taylor's series

2. Expand x^y in powers of (x-1) and (y-1) up to third degree terms

3. Expand $f(x, y) = \tan^{-1}(y/x)$ in powers of $(x - 1)$ and $(y - 1)$ up to and including the second degree terms. Hence compute f(1.1, 0.9).

4. Expand $\dfrac{(x+h)(y+k)}{x+h+y+k}$ in powers of h and k up to second degree terms.

5. Obtain the expansion of $e^x \log(1+y)$ about (0,0)

6. Obtain the expansion of $e^x \sin y$ in the neighborhood of $(1, \pi/4)$

7. Expand sin(xy) in the neighborhood of $(1, \pi/2)$.

8. Expand $x^2 y + 3y - 2$ in the powers of (x-1) and (y+2).

9. Expand $x^2 + 3y^2 - 9x - 9y + 26$ in the powers of (x-1) and (y-2).

10. Expand $(1 + x + y^2)^{1/2}$ at the point $(1, 0)$.